PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Binomial Theorem & Series

SL 1.3, AHL 1.9

ibtuition.sg · free

FORMULAS YOU MUST KNOW

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$$

$$\sum_{r=0}^{n} \binom{n}{r} = 2^n$$

Set $a = b = 1$:

$$\sum_{r=0}^{n} (-1)^r \binom{n}{r} = 0$$

Set $a = 1, b = -1$:

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

(geometric series!)

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$$

$$(1+x)^{1/2} = 1 + \tfrac{x}{2} - \tfrac{x^2}{8} + \tfrac{x^3}{16} - \cdots$$

$$(1+x)^{-1/2} = 1 - \tfrac{x}{2} + \tfrac{3x^2}{8} - \tfrac{5x^3}{16} + \cdots$$

$$\binom{n}{r} = C_r^n = \frac{n!}{r!\,(n-r)!}$$

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r, \qquad n \in \mathbb{N}$$

KEY METHODS

- Use $\binom{n}{r} = \binom{n}{n-r}$ — always pick the smaller r to compute fewer factors.
- r starts at 0, so the first term is T_1 (with $r = 0$). The expansion has exactly $n + 1$ terms.
- The step-back method still works. For $n = \frac{1}{2}$: coefficients are $1, \frac{1}{2}, -\frac{1}{8}, \frac{1}{16}, \ldots$
- $(1-x)^{-1}$ is both the generalised binomial ($n = -1$) AND an infinite geometric series. Recognising this link saves time.
- The IB almost always uses $x = 0.1, 0.01, \frac{1}{9}$, or 0.2. If the question says "suitable value", find which x makes $1 + ax$ equal to your target.
- At the boundary $|x| = 1$: convergence depends on n (beyond IB scope).

EXAM TRAPS TO AVOID

- T_{r+1} has b^r, NOT b^{r+1}.
- $(2x)^3 = 8x^3$, not $2x^3$. Always expand the numerical coefficient.
- $(-b)^r$: even $r \to +$, odd $r \to -$. Don't drop the sign.
- Constant term $\neq$ first term. Set the x-power $= 0$.
- If r comes out fractional or $> n$ in an SL question, the term does not exist.
- "Sum of coefficients": set $x = 1$, not always 2^n (only equal to 2^n when $a = b = 1$).
- Always state validity. $|x| < \dfrac{1}{|a|}$. Costs a guaranteed mark if omitted.

- The IB may ask: "Derive the binomial series using Maclaurin's theorem" — this is the method.