Complex Numbers

FORMULAS YOU MUST KNOW

$z = a + bi,\ a, b \in \mathbb{R},\ i^2 = -1$

General form:

$a = \mathrm{Re}(z)$

Real part:

$b = \mathrm{Im}(z)$

Imaginary part:

$(a + bi) + (c + di) = (a + c) + (b + d)i$

Addition:

$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Multiplication:

$\dfrac{a + bi}{c + di} = \dfrac{(a + bi)(c - di)}{c^2 + d^2}$

Division:

$z^* = a - bi$

Conjugate:

$zz^* = |z|^2 = a^2 + b^2$

Modulus squared:

$z + z^* = 2a \in \mathbb{R}$

Sum:

$z - z^* = 2bi \in i\mathbb{R}$

Difference:

$(zw)^* = z^* w^*$

Product conj.:

$\left(\dfrac{z}{w}\right)^* = \dfrac{z^*}{w^*}$

Quotient conj.:

KEY METHODS

- To divide $a + bi$ by $c + di$: multiply numerator and denominator by $c - di$ (the conjugate of the denominator). The denominator becomes the real number $c^2 + d^2$ — clean and fast.
- On the Argand diagram: $|z_1 - z_2|$ is the geometric distance between the points representing z_1 and z_2. The locus $|z - z_0| = r$ is a circle, centre z_0, radius r .
- To multiply or divide complex numbers, convert to Euler form first — multiply/divide moduli and add/subtract arguments. Far faster than expanding Cartesian brackets, especially when raising to a power.
- Euler's identity: $e^{i\pi} + 1 = 0$ (special case $r = 1$, $\theta = \pi$). Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ is in the IB formula booklet.
- Sum of roots $= -b/a$ and product of roots $= c/a$ (Vieta's). Use these to find the missing real root and

EXAM TRAPS TO AVOID

- $i^2 = -1$, not $+1$. In multiplication, the $bi \cdot di$ term gives $-bd$, not $+bd$ — forgetting this sign is the single most common A1 lost in this section.
- Argument must lie in $(-\pi, \pi]$. Always check the quadrant using the signs of a and b — $\arctan(b/a)$ alone gives the wrong answer for Quadrants 2 and 3.
- $\operatorname{cis}\theta$ is shorthand for $\cos\theta + i\sin\theta$ — not $\cos\theta \cdot i\sin\theta$. Never split it as a product.
- The conjugate root theorem applies only when all coefficients are real. If any coefficient is complex, α^* is not automatically a root.
- De Moivre requires the base to be in the form $\cos\theta + i\sin\theta$ (modulus 1). If $|z| = r \neq 1$, write $z = re^{i\theta}$ first, then $(re^{i\theta})^n = r^n e^{in\theta}$.
- There are exactly n distinct roots — never more, never fewer. Students often stop at 2 or 3 roots for a quartic or quintic. Always count up to $k = n - 1$.

any unknown coefficients without fully expanding the factored form.

- To express $\cos^n \theta$ or $\sin^n \theta$ as a sum of multiple-angle terms: use $z^n + z^{-n} = 2\cos n\theta$, expand $(z + z^{-1})^n$ binomially, then substitute back. This is the standard route on Paper 1.
- Proof by induction for $n \in \mathbb{Z}^+$ is examinable. Know the three steps: base case $n = 1$, assume $n = k$, prove $n = k + 1$ using trig addition formulas.

- When separating $C + iS$ after summing the geometric series, multiply by the complex conjugate of the denominator — not by the denominator itself.