PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Differentiation

SL 5.1–5.8, AHL 5.12–5.15

ibtuition.sg · free

FORMULAS YOU MUST KNOW

$\lim_{x\to a} f(x) = L$ exists iff left-hand limit $=$ right-hand limit.
Limit:

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ (IB: polynomials only).
First principles:

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

$\lim_{x\to\infty} \frac{x^n}{e^x} = 0 \text{ for any } n$

$\frac{d}{dx}[x^n] = nx^{n-1},\ n \in \mathbb{Q}$
Power:

$\frac{d}{dx}[c] = 0$
Constant:

$(f \pm g)' = f' \pm g'$
Sum / diff:

$(cf)' = cf'$
Scalar:

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$
Chain:

$(uv)' = u'v + uv'$
Product:

KEY METHODS

- For first principles with $f(x) = x^n$: expand $(x+h)^n$ using the binomial theorem, cancel h from the numerator, then let $h \to 0$. Every term except the first-order h term vanishes.
- Linear composite shortcut: $[f(ax+b)]' = a \cdot f'(ax+b)$. Just multiply by the inner derivative a.
- $a^x = e^{x\ln a}$, so by chain rule $\frac{d}{dx}[a^x] = e^{x\ln a} \cdot \ln a = a^x \ln a$. Derive it on the spot if you forget.
- $\arctan x$ has no square root in its derivative because it is defined for all $x \in \mathbb{R}$ — the denominator $1 + x^2$ is always positive.
- The gradient at a point on an implicit curve still requires substituting both x AND y coordinates — the formula for $\frac{dy}{dx}$ will contain both variables.
- If the limit is still $\frac{0}{0}$ after one application, apply L'Hôpital again. In IB exams the rule is almost always needed twice.

EXAM TRAPS TO AVOID

- Never substitute $h = 0$ before cancelling h from the numerator — you get $\frac{0}{0}$ and the method fails. Always simplify first.
- $\sin^2 x = (\sin x)^2 \neq \sin(x^2)$. These differentiate completely differently: $(\sin x)^2 \to 2\sin x\cos x$; $\sin(x^2) \to 2x\cos(x^2)$.
- Quotient rule order: it is $u'v - uv'$, NOT $uv' - u'v$. Subtraction is not commutative.
- $\arcsin'$ and $\arccos'$ are negatives of each other. Don't mix them up under exam pressure.
- $f''(a) = 0$ does NOT automatically mean a point of inflexion. You must verify f'' changes sign at $x = a$. Classic counter-example: $f(x) = x^4$ has $f''(0) = 0$ but $(0,0)$ is a minimum, not an inflexion.
- The most common mark lost: forgetting to multiply by $\frac{dy}{dx}$ when differentiating a y-term. $\frac{d}{dx}[y^2] =$

- For optimisation, if the domain is bounded (e.g. $0 < x < 4$), always evaluate at the endpoints too — the global maximum may be there, not at the stationary point.

$2y\dfrac{dy}{dx}$, NOT just $2y$.

- L'Hôpital cannot be used if the form is NOT indeterminate — e.g. $\frac{\sin x}{x+1}$ at $x = 0$ gives $\frac{0}{1} = 0$ directly. Applying L'Hôpital to a non-indeterminate form gives the wrong answer.