PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Integration

SL 5.5, 5.10–5.11, AHL 5.15–5.17

ibtuition.sg · free

FORMULAS YOU MUST KNOW

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$$

$$\int u\,dv = uv - \int v\,du$$

$f(x)$	$x^n \quad (n \neq -1)$
$\frac{1}{x}$	e^x
a^x	$\sin x$
$\cos x$	$\tan x$
$\sec^2 x$	$\csc^2 x$

KEY METHODS

- For $\int \frac{1}{x^2+bx+c}\,dx$: complete the square first to get $(x+p)^2+q^2$, then use $\frac{1}{q}\arctan\frac{x+p}{q}$.
- For $\int \frac{f'(x)}{f(x)}\,dx$: any fraction where the top is (a multiple of) the derivative of the bottom integrates to $\ln|\text{bottom}|$. Check this pattern first before reaching for substitution.
- If $g'(x)$ is NOT present in the integrand (not even as a scaled multiple), inspection will fail — switch to formal substitution instead.
- After substituting, check no x remains. If an x is left over, rearrange $u = g(x)$ to express x in terms of u.
- $\int \ln x\,dx$: treat as $\int \ln x \cdot 1\,dx$ with $u = \ln x$, $dv = dx$. The 1 is the invisible dv.
- Substitute $x = a$ (a root of a linear factor) to find the corresponding constant in one step — no simultaneous equations required.
- Odd function on a symmetric interval: $\int_{-a}^{a} f = 0$ instantly. Even function: $\int_{-a}^{a} f = 2\int_0^a f$. Check this

EXAM TRAPS TO AVOID

- $\int \frac{1}{x}\,dx = \ln|x| + C$ — the absolute value bars are required and cost a mark every time they are omitted.
- $\int 6x(x^2+1)^4\,dx$: don't multiply $6x$ into the answer. Factor as $3 \cdot 2x$ first, then apply the power rule: answer is $\frac{3(x^2+1)^5}{5} + C$.
- Forgetting to change the limits in a definite integral after substitution. Continuing with the original x-values is wrong and loses every accuracy mark in the question.
- Cyclic IBP — swapping your choice of u in round 2 collapses to the trivial $0 = 0$. Always keep the same function as u in both rounds.
- If $\deg(P) \geq \deg(Q)$, do polynomial long division first before decomposing. Skipping this step gives an incorrect partial-fraction setup.
- $\int_a^b f\,dx$ gives signed area — it can be negative. "Find the area" always means geometric (positive). Never integrate directly when f dips below the axis.

before starting any integral on symmetric limits.

- $(f-g)^2 \neq f^2 - g^2$. Square each radius separately: $\pi \int ([f]^2 - [g]^2)\, dx$. Squaring the difference is wrong — a very common exam mistake.