PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Maclaurin Series & Limits

AHL 5.19–5.20

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FORMULAS YOU MUST KNOW

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

e^x:

$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ (odd po

$\sin x$:

$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ (even powers only)

$\cos x$:

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$\ln(1+x)$:

$1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$

$(1+x)^p$:

$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

$\arctan x$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} =$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$$

$$f(x) = \sum_{n=0}^{\infty}$$

$$\ln(1+\sin x) = \left(x - \frac{x^3}{6}\right) - \frac{1}{2}\left(x^2 - \frac{x^4}{3}\right) + \frac{x^3}{3} - \frac{x^4}{4} + O(x^5).$$

$$\frac{\ln(1+\sin x) - x}{x^2} = \frac{-\frac{x^2}{2} + \frac{x^3}{6} -}{x^2}$$

KEY METHODS

- For $\sin x$: the odd-power pattern means $f^{(n)}(0) = 0$ for every even n — no even-power terms ever appear. Same logic: $\cos x$ has no odd-power terms. Use this to save half the differentiation work in "find from first principles" questions.
- To find the Maclaurin series for $\sec x$: write $\sec x = (1-u)^{-1}$ where $u = 1 - \cos x = \frac{x^2}{2} - \frac{x^4}{24} + \cdots$, then expand $(1-u)^{-1} = 1 + u + u^2 + \cdots$ and collect up to the required degree.
- For ∞/∞ involving exponentials, try dividing the numerator and denominator by the dominant exponential before reaching for L'Hôpital — usually faster, and avoids repeated differentiation.

EXAM TRAPS TO AVOID

- Writing $\ln(1-x) = x - \frac{x^2}{2} + \cdots$ with the wrong sign. Always substitute $-x$ into $\ln(1+x)$: result is $-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$ (every term negative).
- When composing $e^{f(x)}$, students forget to expand u^2 and u^3 terms. For $e^{\sin x}$: $u^2 = (x - \frac{x^3}{6})^2 = x^2 - \frac{x^4}{3} + \cdots$ — the x^4 contribution matters if you need the x^4 coefficient.
- Applying L'Hôpital when the form is NOT indeterminate. For example, $\lim_{x\to 2} \frac{x^2-4}{x-2}$ factorises as $(x+2)$ directly. Using L'Hôpital here gives the right

- When both methods apply, the series method is almost always faster at $x = 0$. Two applications of L'Hôpital to find $\lim_{x\to 0} \dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$ takes 6 lines; the series gives it in 2.
- To integrate $e^{x^2}\sin(x^2)$: first find the series for $e^x \sin x = x + x^2 + \dfrac{x^3}{3} + \cdots$, then substitute $x \to x^2$ to get $e^{x^2}\sin(x^2) = x^2 + x^4 + \dfrac{x^6}{3} + \cdots$, then integrate term by term.
- Setting $x = 1$ in $\arctan x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \cdots$ gives the Leibniz formula $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \cdots$, so $\pi = 4(1 - \dfrac{1}{3} + \dfrac{1}{5} - \cdots)$. An IB-favourite "show that" result.
- To verify a Maclaurin series found by first principles: also expand using composition (if applicable) and check that the first few coefficients agree. Disagreement signals an error in one of the derivatives.

answer numerically but misses the factorisation method mark.

- Forgetting to carry enough terms in the series. For $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$, you need the x^3 terms of both $\tan x$ and $\sin x$ — stopping at x^2 leaves $0/0$ with no answer.
- Trusting the Maclaurin approximation over a large interval. The series for $\cos^2 x$ integrated over $[0, \pi]$ gives a poor approximation because $x = \pi/2 \approx 1.57$ is not small. Always check that the upper limit is well within the interval of fast convergence.
- Using $(1+x)^p$ when the expression is $(a+bx)^p$: factor first as $a^p\left(1 + \dfrac{b}{a}x\right)^p$, then apply the series with x replaced by $\dfrac{b}{a}x$. The validity condition becomes $\left|\dfrac{b}{a}x\right| < 1$.
- When differentiating an ODE to find y'', students forget to differentiate the y-terms via the chain rule. For $\dfrac{dy}{dx} = y\cos x$: $\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\cos x - y\sin x$ — both terms involve y' implicitly.