PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Polynomials

SL 2.5–2.7, AHL 2.12–2.16

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FORMULAS YOU MUST KNOW

$\Delta = b^2 - 4ac$

Δ:

$P(a)$ when dividing by $(x-a)$, so $P(x) = (x-a)\,Q(x) + P(a)$

Remainder:

$P(x) = D(x) \cdot Q(x) + R(x), \quad \deg(R) < \deg(D)$

$\sum r_i = -\frac{a_{n-1}}{a_n}$

Sum:

$\prod r_i = (-1)^n \frac{a_0}{a_n}$

Product:

$$P(a) = 0 \qquad \text{AND} \qquad P'(a) = 0$$

$a_n > 0$

$(x-a)$

$(x-a)^2$

$(x-a)^3$

$2k$ (even)

$2k+1$ (odd)

KEY METHODS

- At most $n-1$ turning points and n real roots for degree n. Count x-intercepts on a sketch to narrow down the degree immediately.
- For real-coefficient quartics, the number of real roots is always 0, 2, or 4 (never 1 or 3) — complex roots come in conjugate pairs.
- As soon as you see a complex root in a real polynomial, write the conjugate before doing anything else. This is always the first line of working.
- Testing $P(-1)$ and $P(1)$ first saves time — both are mental arithmetic. Only then try $\pm 2, \pm 3$, etc.
- Coefficient comparison is faster than long division when you already know the form of the quotient. Write $P(x) = D(x)(Ax^k + \cdots)$ and match powers of x from highest down.
- For a quartic with no obvious rational root: try to express it as $(x^2+px+q)(x^2+rx+s)$ and solve for p, q, r, s via coefficient comparison. Only four equations needed.

EXAM TRAPS TO AVOID

- A double root means the graph bounces — do not draw it crossing. Check multiplicity before sketching.
- The real quadratic factor is $(x-a)^2 + b^2$, not $(x-a)^2 - b^2$. The sign of b^2 is always positive.
- The Conjugate Root Theorem only applies when all coefficients are real. If the polynomial has complex coefficients, this does not hold.
- When the question says "exactly divisible by $(x-a)^2$" on Paper 1, you must differentiate and set $P'(a) = 0$. Many students only use $P(a) = 0$ and lose half the marks.
- Dividing by a quadratic gives a remainder of the form $mx + c$ (linear), not a constant. Many students write a constant remainder and lose the method mark immediately.
- "Fully factorise" means over $\mathbb{R}$ unless the question says "over $\mathbb{C}$". Leave irreducible quadratics intact when factorising over $\mathbb{R}$.

- For "find the equation with roots $f(\alpha), f(\beta)$": compute the new sum and new product using Vieta's on the original, then write $x^2 - (\text{new sum})x + (\text{new product}) = 0$.

- Sum of roots $= -b/a$, NOT $+b/a$. The minus sign is always there. If your sum looks suspiciously like b/a, you have the wrong sign.