PHOTON ACADEMY · IB MATH AA HL CHEATSHEET

Proof by Mathematical Induction

AHL 1.15

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FORMULAS YOU MUST KNOW

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

$$\sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$$

$$\sum_{r=1}^{n} \binom{r}{1} = \binom{n+1}{2}$$

$$\sum_{r=1}^{k+1} f(r) = \underbrace{\sum_{r=1}^{k} f(r)}_{\text{apply IH}=S(k)} + \underbrace{f(k+1)}_{\text{next term}} = S(k) + f(k+1) \stackrel{?}{=} S(k+1)$$

$$\frac{k(k+1)}{2} + (k+1) = (k+1$$

$$2\sin\theta\cos\phi = \sin(\phi+\theta) - \sin(\phi-\theta)$$

$$(\cos\theta + i\sin\theta)^{k+1} = (\cos k$$

$$f^{(k+1)}(x) = \frac{d}{dx}\left[f^{(k)}(x)\right] = \frac{d}{dx}[\text{IH formula}]$$

KEY METHODS

- Always compute both sides of the base case explicitly — even if it looks obvious. The IB awards R1 only when both sides are shown. Writing "LHS $= \frac{1}{2}$" without showing RHS $= \frac{1}{2}$ loses the mark.
- For the final R1 (conclusion): the IB requires at least 4 of the previous 6 marks. So even if your algebra has gaps, write the conclusion sentence every time — you can still earn it.
- Before starting the algebra, write on your paper: Next term: $f(k+1) = \ldots$ Target: $S(k+1) = \ldots$. This tells you exactly where you're heading. Never start manipulating without knowing the target.
- For alternating sums like $1 - 4 + 9 - \ldots + (-1)^{n+1}n^2 = (-1)^{n+1}\frac{n(n+1)}{2}$: the next term is $(-1)^{(k+1)+1}(k+1)^2 = (-1)^{k+2}(k+1)^2 =$

EXAM TRAPS TO AVOID

- Never write "Let $n = k$" or "Assume $n = k$ is true" — the IB mark scheme explicitly states: "Do not award M1 for statements such as 'let $n = k$'." You must write the formula.
- Omitting the conclusion sentence always costs R1, even if every other step is perfect. The IB awards this R1 only when at least 4 of the prior 6 marks have been earned.
- The most common factorial error: for the sum $\sum_{r=1}^{n} \frac{r}{(r+1)!}$, when $r = k+1$, the denominator is $(\mathbf{k+2})!$, not $(k+1)!$. The $r+1$ in the denominator means you substitute $r = k+1$ to get $(k+1)+1 = k+2$.
- When writing the IH for divisibility, don't write "$f(k)$ is divisible by d" — write "$f(k) = dm$ for some $m \in \mathbb{Z}$".

$(-1)^k(k+1)^2$. Always simplify the sign before proceeding.

- For combinatorics sums involving $\binom{r}{1}$, the key identity is Pascal's Rule: $\binom{k+1}{2}+\binom{k+1}{1}=\binom{k+2}{2}$. This comes directly from the $\binom{n}{r}$ definition and is the only algebra needed.
- At the start of the inductive step, always write: "Let $f(k)=dm$ for some $m\in\mathbb{Z}$." Then at the end write: "$=d(\ldots)$ where $\ldots\in\mathbb{Z}$, so divisible by d." These two sentences bookend your argument.
- For inequality induction, the IB often gives you a helper result in part (a) to use in part (b). The helper result is almost always used as the second link in the chain. Write both links on separate lines with their justification.

The algebraic form is needed for the inductive step. Without it you can't complete the algebra.

- Threshold inequalities (e.g. "for all $n\geq 5$"): the base case must be $n=5$, not $n=1$. Also, within the inductive step, you must use $k\geq 5$ when making estimates. For example, saying $(k-1)^2\geq 16$ requires $k\geq 5$.
- When adding the common denominator in trig induction, don't forget to multiply the $(k+1)$-th term numerator and denominator by $2\sin\theta$. Write: $\cos(2k+1)\theta=\dfrac{2\sin\theta\cos(2k+1)\theta}{2\sin\theta}$.
- In the base case for derivative proofs, two A1 marks are awarded: one for computing $f'(x)$ correctly, and one for verifying it matches the formula at $n=1$. Many students compute the derivative but forget to substitute $n=1$ into the RHS and compare.