Proof

FORMULAS YOU MUST KNOW

$n = 2k$, $k \in \mathbb{Z}$	$n = 2k + 1$, $k \in \mathbb{Z}$
$n = mk$, $k \in \mathbb{Z}$	$r = p/q$, $\gcd(p, q) = 1$, $q \neq 0$
$p^2 - 8q - 11 \neq 0$ for $p, q \in \mathbb{Z}$	$n = 40$: $40^2 + 40 + 41 = 41^2$
$n = 41$: $41^2 + 41^2 + 41 = 41 \times 83$	$n = 4$: $16 + 4 + 5 = 25 = 5^2$
$n = 4$: $2^4 - 1 = 15 = 3 \times 5$	$(m + n)^2 = m^2 + n^2$
$n = 30$	$x = 1, y = 3: 1 + 9 = 10$

KEY METHODS

- Express, don't just state. Always write out the algebraic form. If n is even, write $n = 2k$ where $k \in \mathbb{Z}$. If n is odd, write $n = 2k + 1$. Then substitute and compute.
- Product of consecutives is always even: $n(n + 1)$ — one of $n, n + 1$ is always even, so the product is even.
- Divisible by m means $= m \times (\text{integer})$. To prove E is divisible by 12, show $E = 12k$ for some $k \in \mathbb{Z}$, or factor as $E = 12 \times (\ldots)$ where the bracket is an integer.
- Difference of squares shortcut: $(a + b)^2 - (a - b)^2 = 4ab$ always. Use this to spot divisibility quickly.
- The "hence" signal. When a question says "hence prove", the previous part has set up exactly what you need. Use the result you just proved — don't rederive it. This is tested every year.
- For LHS = RHS proofs, always expand the more complicated side first (or expand both sides towards a common middle). Never "work from both ends simultaneously" — the IB expects a single directed chain.

EXAM TRAPS TO AVOID

- "n is even, so n^2 is even" is NOT a proof. You must show it: $n = 2k \Rightarrow n^2 = 4k^2 = 2(2k^2)$, divisible by 2. ✓ Just asserting "even × even = even" earns 0 marks.
- Consecutive integers — choose your representation wisely. For the sum of 3 consecutives, $n - 1, n, n + 1$ gives sum $= 3n$ immediately. Using $n, n + 1, n + 2$ gives $3n + 3 = 3(n + 1)$ — still works but messier.
- Don't assume what you're proving. In an identity proof, you cannot start with "$\text{LHS} = \text{RHS}$" and then manipulate — that assumes the result. Always start with one side and arrive at the other.
- "Let" earns M0 — use "Assume" or "Suppose". Wrong: "Let $\sqrt{3} = p/q$..." — this defines, it doesn't contradict. Correct: "Assume for contradiction that $\sqrt{3}$ is rational, so $\sqrt{3} = p/q$ where $p, q \in \mathbb{Z}$, $q \neq 0$, $\gcd(p, q) = 1$." The IB mark scheme explicitly awards M0 for "let", "consider", or "given that".
- Wrong inequality direction. If proving $f \geq 4$, assume $f < \mathbf{4}$ (strict), not $f \leq 4$. If proving $f > 4$, assume $f \leq \mathbf{4}$. Wrong direction $\Rightarrow$ M0 on the assumption mark.
- No contradiction statement. Just arriving at "$(2x - 1)^2 < 0$" is not enough. You must write: "This is a

- Sum of squares of consecutive odds: $(2n-1)^2 + (2n+1)^2 = 8n^2 + 2 = 2(4n^2+1)$ is always even. Know this pattern.

contradiction since $(2x-1)^2 \geq 0$ for all $x \in \mathbb{R}$." Without this, the final R1 is lost.

- Not justifying multiplication of an inequality. When $0 < x < 1$, before multiplying by $x(1-x)$, you must state: "Since $x > 0$ and $1 - x > 0$, $x(1-x) > 0$, so the inequality direction is preserved." The examiner awards a specific R1 for this.