Systems of Linear Equations

FORMULAS YOU MUST KNOW

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases} \longrightarrow \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 7 \\ 1 & 0 & 4 & -1 \\ 2 & -3 & 0 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow (x, y, z) = (1, 2, 3).$$

$$\left[\begin{array}{ccc|c} 1 & 0 & a & p \\ 0 & 1 & b & q \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$(x, y, z) = (p - at,\ q - bt,\ t).$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ q \\ 0 \end{pmatrix} + t\begin{pmatrix} -a \\ -b \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{17}{4} \\ 0 & 1 & -\frac{3}{2} & \frac{3}{4} \end{array}\right]$$

KEY METHODS

- If the (1,1) entry is 0 or large, swap rows to bring the simplest leading entry to the top — e.g.\ swap $R_1 \leftrightarrow R_3$ if R_3 starts with 1. Cleaner numbers all the way down.
- "Does not have a unique solution" includes both infinite solutions ($k = 0$) and no solution ($k \neq 0$). You must check the right-hand side of the bottom row after setting $g(a) = 0$.

EXAM TRAPS TO AVOID

- If the (2,1) entry is -2, the operation is $R_2 \to R_2 - (-2)R_1 = R_2 + 2R_1$. The multiplier is (entry) $\div$ (pivot) — always check the sign before writing the operation.
- Only the target row changes. If you write $R_2 \to R_2 - kR_1$, R_1 remains exactly as before. Never alter R_1 when targeting R_2.
- Never just write "infinite solutions". The IB always awards marks for the explicit parametric form: $x = \ldots,\ y = \ldots,\ z = t,\ t \in \mathbb{R}$.