Vectors & Planes

FORMULAS YOU MUST KNOW

$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
Magnitude:

$\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}, \quad |\hat{\mathbf{v}}| = 1$
Unit vector:

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
Displacement:

$|AB| = |\mathbf{b} - \mathbf{a}|$
Distance:

$\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$
Midpoint:

$\overrightarrow{OP} = \dfrac{n\mathbf{a} + m\mathbf{b}}{m + n}$
Ratio $m : n$:

$\mathbf{a} = k\mathbf{b}, \quad k \neq 0$
Parallel:

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$
Algebraic:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$
Geometric:

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}, \quad \theta \in [0^\circ, 180^\circ]$
Angle:

$\mathbf{a} \cdot \mathbf{b} = 0$
Perpendicular:

$|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a}||\mathbf{b}|$
Parallel:

KEY METHODS

- Parallelogram OABC: always $\overrightarrow{AB} = \mathbf{c}$, $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, $\overrightarrow{CB} = \mathbf{a}$. Set these up before anything else.
- Regular hexagon ABCDEF with $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$: $\overrightarrow{OC} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{OD} = -\mathbf{a}$, $\overrightarrow{OE} = -\mathbf{b}$, $\overrightarrow{OF} = \mathbf{a} - \mathbf{b}$.
- Proof shortcut: replace $\mathbf{a} \cdot \mathbf{b}$ with $|\mathbf{a}||\mathbf{b}| \cos\theta$ and $|\mathbf{a} \times \mathbf{b}|$ with $|\mathbf{a}||\mathbf{b}| \sin\theta$, then $\sin^2\theta + \cos^2\theta = 1$ kills the whole thing in two lines.
- Parallelogram with $|\mathbf{c}| = 2|\mathbf{a}|$: substitute $\mathbf{a} \cdot \mathbf{c} = 2|\mathbf{a}|^2 \cos\theta$ directly — don't try to expand in components.
- $\theta \in [0^\circ, 180^\circ]$ always — the dot product always gives the unique angle between two vectors, never an ambiguous result.
- After computing $\mathbf{a} \times \mathbf{b}$: immediately verify with $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} \stackrel{?}{=} 0$ and $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} \stackrel{?}{=} 0$. Takes 20

EXAM TRAPS TO AVOID

- Collinear requires two things: (1) vectors are parallel, AND (2) they share a point. Missing either loses a mark.
- After showing $\overrightarrow{MN} = k\overrightarrow{AB}$ you must write: "Since $\overrightarrow{MN}$ is a scalar multiple of $\overrightarrow{AB}$, MN is parallel to AB." This reasoning sentence earns the R mark.
- Order matters: $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$ — the sign flips. Always state which order you used, especially in "show" questions.
- If $l = 0$, you cannot write $\dfrac{x - x_0}{0}$ — it is undefined. Instead write $x = x_0$ separately alongside the other two ratios.
- The most common mark lost in this topic: skipping Step 4. Every single time, check the 3rd equation. Solving 2 equations alone is never sufficient to confirm intersection.

seconds and catches all arithmetic errors before they cost you marks.

- Kinematics speed: $\mathbf{r} = \mathbf{a} + 4t\mathbf{b}$ has speed $= 4|\mathbf{b}|$, not $|\mathbf{b}|$. The scalar factor before t multiplies the whole velocity vector.

- The perpendicular distance formula is NOT in the IB data booklet. Never try to memorise $d = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$. Use the line method: write the line through your point with direction $\mathbf{n}$, find where it meets the plane, then compute the distance. This always works.

- Line–Plane uses $\sin$, not $\cos$. The angle α is measured from the plane surface to the line, not from the normal. The complement of α (i.e. $90^\circ - \alpha$) is the angle between the line and the normal — that one uses $\cos$. This is the #2 most-lost mark in this topic.