Topic A.4

RIGID BODY MECHANICS

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered	**Exam Relevance**
Torque & Rotational Equilibrium	HL only — P1 & P2
Angular Kinematics (rot. SUVAT)	α, ω, θ problems
Moment of Inertia & $\tau = I\alpha$	Rotational Newton's 2nd law
Angular Momentum & Conservation	Conservation of L (skater)
Rotational KE & Rolling	Rolling down inclines

The Big Picture — Rigid Body = Rotational Twin of Linear Mechanics

RIGID BODY MECHANICS
every linear tool has a rotational twin: $F \to \tau$, $m \to I$, $v \to \omega$, $p \to L$

- **Torque**
  $\tau = Fr\sin\theta$
  $= F \times d_\perp$
  $\sum\tau = 0$ (equil.)
- **Angular Kinematics**
  θ, ω, α
  rotational SUVAT
  $v = r\omega$
- **Moment of Inertia**
  $I = \sum mr^2$
  (shape values *given* in exam)
- **Dynamics**
  $\tau = I\alpha$
  $L = I\omega$
  $\Delta L = \tau\Delta t$
- **Rotational Energy**
  $\frac{1}{2}I\omega^2$
  rolling: $\frac{1}{2}Mv^2$
  $+\frac{1}{2}I\omega^2$

§1 — Torque

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Core Formulae

Torque: $\tau = Fr\sin\theta$

Rotational equilibrium: $\sum\tau = 0$

Torque = : Force $\times$ perpendicular distance from pivot

Couple: $\tau = Fd$ (two equal antiparallel forces; same about any axis)

Static equilibrium: $\sum F = 0$ AND $\sum\tau = 0$

TRICK **Two ways to find torque:** $\tau = Fr\sin\theta$ (full formula) OR $\tau = F \times d_\perp$ with $d_\perp = r\sin\theta$ the perpendicular distance from the pivot to the line of action. Draw the line of action and drop a perpendicular — usually the fastest route.

TRAP θ is the angle between the force vector and the lever arm (line from pivot to point of force application), NOT the angle between the force and any surface.

NOTE Maximum torque when $\theta = 90°$ ($\tau = Fr$). Zero torque when $\theta = 0°$ (force passes through pivot).

Rotational Equilibrium — Taking Moments

Choose a pivot at an **unknown force** to eliminate it from the equation. Then: $\sum\tau_{\text{CCW}} = \sum\tau_{\text{CW}}$.

NOTE **Centre of gravity:** whole weight acts at one point — it exerts *no* moment about that point. A plank balances only if the pivot is at its CoG. Suspend a body freely and its CoG hangs directly below the suspension point.

NOTE **Units:** torque is in N m but is never called a joule. Only *energy* in N m is renamed the joule.

TRICK **Three non-parallel forces** in equilibrium must all pass through a single point (concurrent). Great for finding the direction of an unknown reaction (e.g. a ladder on a smooth wall — the wall gives only a horizontal normal force, so friction at the ground is essential).

FROM THE PHOTON QUESTION BANK

Examiners test static-equilibrium set-ups: draw *all* forces, note that a **smooth** wall/surface gives only a normal reaction, take moments about an unknown force to remove it, and quote a couple's torque as Fd (axis-independent).

§2 — Angular Kinematics (SUVAT)

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SUVAT Analogy

Linear	Rotational	Missing
$v = u + at$	$\omega_f = \omega_i + \alpha t$	$\Delta\theta$
$s = \frac{1}{2}(u+v)t$	$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)t$	α
$s = ut + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	ω_f
$v^2 = u^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	t

Links to linear: $s = r\theta \quad v = r\omega \quad a_t = r\alpha \quad a_c = r\omega^2$
Total accel. of a rim point: $a = \sqrt{a_c^2 + a_t^2} = r\sqrt{\omega^4 + \alpha^2}$

TRAP $\Delta\theta$ must be in **radians**. If the problem gives revolutions n, convert: $\Delta\theta = 2\pi n$. For a period, $\omega = 2\pi/T$.

TRICK Use $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$ when time is not given and not needed.

NOTE **Rolling wheel** = translation $(+v)$ + rotation $(r\omega)$. Bottom (contact) point: 0; top: $2v$; side: $\sqrt{2}\,v$ at 45°.

NOTE **Graphs:** gradient of θ–t is ω; gradient of ω–t is α and its *area* is $\Delta\theta$; *area* under τ–t is angular impulse ΔL.

§3 — Moment of Inertia

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Key Formula and Reference Table

Definition: $I = \sum m_i r_i^2$

Shape	Axis	I
Solid disc/cylinder	Central, along axis	$\frac{1}{2}MR^2$
Thin ring	Central, along axis	MR^2
Thin ring	Through a diameter	$\frac{1}{2}MR^2$
Solid sphere	Through centre	$\frac{2}{5}MR^2$
Thin rod	Through centre, $\perp$	$\frac{1}{12}ML^2$
Thin rod	Through end, $\perp$	$\frac{1}{3}ML^2$
Rect. lamina $l \times h$	Central, $\perp$ plane	$\frac{1}{12}M(l^2+h^2)$

NOTE I depends on both total mass AND how the mass is distributed relative to the axis. More mass further from the axis $\Rightarrow$ larger I. Derivation: $F = mr\alpha \Rightarrow \tau = mr^2\alpha$, sum over particles $\Rightarrow \tau = I\alpha$ with $I = \sum mr^2$.

FROM THE PHOTON QUESTION BANK

Examiners test $I = \sum mr^2$ for point masses — use the *perpendicular* distance to the axis (for two masses on a rod spun about its centre, that is **half** the rod length, not the full length).

NOTE The I values in the table above are **given in the exam** (data booklet / question) — you do not need to memorise them. You DO need to select the right one and use it.

TRAP I changes if the axis changes, even for the same object. Always state which axis is being used.

TRICK For a system of discrete masses: just sum mr^2 for each piece. $I = m_1 r_1^2 + m_2 r_2^2 + \ldots$

§4 — Newton's 2nd Law for Rotation

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Core Formula

Rotational N2L: $\tau = I\alpha \quad \Longleftrightarrow \quad F = ma$

Angular acceleration: $\alpha = \dfrac{\tau_{\text{net}}}{I}$

TRICK **Two ways to write it:** $\tau = I\alpha$ when I is constant (get α directly); $\tau = \Delta L/\Delta t$ when I changes or a torque acts over time. Same law.

TRICK For a pulley problem: write $F = ma$ for each block and $\tau = I\alpha$ for the pulley. Use $a = r\alpha$ to link them. Solve simultaneously.

TRAP When a net torque of zero does NOT mean no torques act — it means the torques balance. Use $\tau = I\alpha$ only for the **net** torque.

§5 — Angular Momentum & Impulse

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Core Formulae

Angular momentum: $L = I\omega$

Conservation: $I_1\omega_1 = I_2\omega_2$ (no external torque)

Angular impulse: $\Delta L = \tau\,\Delta t$

TRICK **Two ways to find** L: $L = I\omega$ for a rigid body (disc/rod/sphere); $L = mvr$ for a point mass a perpendicular distance r from the axis (ball on a string, planet). Same thing: $I = mr^2$, $v = \omega r$.

TRICK Conservation of angular momentum problems: set up $I_1\omega_1 = I_2\omega_2$. List what is given, what changes, and solve for the unknown.

TRAP Angular impulse units are **N m s** (newton metre seconds), NOT N m (torque) or N s (linear impulse). This is a common MCQ trap.

NOTE When a skater pulls in their arms, I decreases $\Rightarrow$ ω increases. KE increases too — the extra energy comes from the work done by the skater's muscles. Two wheels coupling: L conserved ($I_1\omega_1 = (I_1{+}I_2)\omega_f$) but KE is lost (rotational "inelastic collision").

FROM THE PHOTON QUESTION BANK

Examiners test conservation of L ("no external torque $\Rightarrow$ $I_1\omega_1 = I_2\omega_2$", KE *not* conserved), the unit of L (kg m^2/s = N m s, distinct from linear momentum and energy), and $L = \sqrt{2IE_K}$ to compare systems of equal rotational KE.

§6 — Rotational Kinetic Energy

Topic A.4 HL

Core Formulae

Work done by torque: $W = \tau\theta$ (power $P = \tau\omega$)

Work–energy theorem: $W = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

Rotational KE: $E_K = \frac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$

Rolling (no slip): $v = r\omega \quad \Rightarrow \quad E_{K,\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

Solid cylinder rolling: $E_{K,\text{total}} = \frac{3}{4}Mv^2$ (ratio $2:1$ trans:rot)

TRICK For rolling down a slope: use energy conservation $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. Substitute $\omega = v/r$ and solve for v. No need for torque equations.

TRAP A hollow object (ring, hollow sphere) has greater I than a solid one of the same mass and radius $\Rightarrow$ it rolls more slowly down a slope (more energy goes to rotation).

NOTE $E_K = \dfrac{L^2}{2I}$ is extremely useful when comparing objects with the same angular momentum or same rotational KE.

FROM THE PHOTON QUESTION BANK

Examiners test rolling-without-slipping energy: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ with $v = R\omega$ (bigger $I/MR^2 \Rightarrow$ slower at the bottom), plus power in a shaft $P = \tau\omega$.

Quick-Check: Linear vs. Rotational Analogies

Linear	Rotational
F (force, N)	τ (torque, N m)
m (mass, kg)	I (moment of inertia, kg m^2)
a (accel., m s^{-2})	α (angular accel., rad s^{-2})
v (velocity, m s^{-1})	ω (angular velocity, rad s^{-1})
$p = mv$ (momentum, N s)	$L = I\omega$ (ang. momentum, N m s)
$F\Delta t = \Delta p$	$\tau\Delta t = \Delta L$
$W = Fs$	$W = \tau\theta$
$E_K = \frac{1}{2}mv^2$	$E_K = \frac{1}{2}I\omega^2$
$\sum F = 0$ (transl. equil.)	$\sum \tau = 0$ (rot. equil.)