Topic A.5

RELATIVITY

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered	Exam Relevance
Galilean vs Einstein Relativity	HL only — P1 & P2
Lorentz Factor & Time Dilation	$\Delta t = \gamma\,\Delta t_0$ (proper time)
Length Contraction & Lorentz Transforms	$L = L_0/\gamma$ (proper length)
Velocity Addition & Space-Time Interval	Invariant space-time interval
Space-Time Diagrams & Muon Decay	Muon decay & the twin paradox

The Big Picture — Two Postulates, and Everything Follows

SPECIAL RELATIVITY
2 postulates $\Rightarrow$ time & space are relative, but c and s^2 are absolute

- **Postulates & γ**: laws same; c constant; $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$
- **Time Dilation**: $\Delta t = \gamma \Delta t_0$; moving clocks run slow
- **Length Contraction**: $L = L_0/\gamma$; moving lengths shrink
- **Lorentz & Velocity**: $x' = \gamma(x - vt)$; $u' = \dfrac{u - v}{1 - uv/c^2}$
- **Invariants & Diagrams**: $s^2 = (c\Delta t)^2 - \Delta x^2$; simultaneity; Minkowski

§1 — Galilean Relativity

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Galilean Transformations

S′ moves at $+v$ relative to S in x-direction:

$$x' = x - vt$$
$$t' = t \quad \text{(absolute time)}$$
$$u' = u - v \quad \text{(velocity addition)}$$

Key Concept

Inertial frame: non-accelerating frame. Newton's laws hold unchanged.

Galilean Relativity: Newton's laws same in all inertial frames. No mechanical experiment can detect uniform motion.

TRICK At everyday speeds ($v \ll c$), Galilean addition is accurate. Only near c do relativistic corrections matter.

NOTE The two postulates of SR: (1) Laws of physics same in all inertial frames; (2) Speed of light c is the same for all observers.

TRAP Galilean addition predicts light speed changes with observer motion. This **contradicts** experiment — use Lorentz instead.

§2 — The Lorentz Factor

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Lorentz Factor γ

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \geq 1$$

Common values:

v/c	γ
0.6	1.25
0.8	$5/3 \approx 1.667$
0.9	2.29
0.99	7.09

Time Dilation

$$\Delta t = \gamma\, \Delta t_0 \quad (\Delta t \geq \Delta t_0)$$

Δt_0 = **proper time**: measured by clock *present at both events* (shortest time).
Δt = dilated time measured by stationary observer.

Moving clocks run **slow**.

TRAP Proper time belongs to the *moving* object (the one present at both events). Students often assign it to the "stationary" observer — this is wrong.

TRICK Memorise: $v = 0.6c \Rightarrow \gamma = 1.25$; $v = 0.8c \Rightarrow \gamma = 5/3$. These appear in almost every IB paper.

FROM THE PHOTON QUESTION BANK

Decide which single clock is at *both* events (that reads Δt_0, the shorter time), then **multi-**

ply by γ for the observed time. Muon lifetime is the classic prompt.

§3 — Length Contraction & Lorentz Transforms

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Length Contraction

$$L = \frac{L_0}{\gamma} \quad (L \le L_0)$$

L_0 = **proper length**: measured in frame where object is at rest (longest length).
Only dimension **parallel** to motion contracts. y, z unchanged.

TRICK Muon duality: in Earth frame, use time dilation ($\Delta t = \gamma\tau_0$). In muon frame, use length contraction ($L = L_0/\gamma$). Both give the same physical prediction.

TRICK **Two ways to derive** time dilation & length contraction: the light-clock thought experiment (physical) OR the Lorentz transforms — set $x' = 0$ for time dilation, $\Delta t = 0$ for length contraction.

Lorentz Transformations

S′ moves at $+v$ relative to S:

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$

Inverse (S′ $\to$ S):

$$x = \gamma(x' + vt')$$
$$t = \gamma(t' + vx'/c^2)$$

FROM THE PHOTON QUESTION BANK

Proper length L_0 is measured in the object's *rest* frame and is the *longest*; a moving observer **divides** by γ. Only the dimension *along* the motion contracts.

§4 — Velocity Addition & Space-Time Interval

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Relativistic Velocity Addition

Object moves at u in S; S′ moves at v relative to S:

$$u' = \frac{u - v}{1 - uv/c^2}$$

Inverse: $u = \dfrac{u' + v}{1 + u'v/c^2}$

Check: if $u = c$, then $u' = c$ always. ✓

Space-Time Interval (Invariant)

$$(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2$$

Same value in ALL frames (invariant).
> 0: timelike (causal link possible)
$= 0$: lightlike (connected by light)
< 0: spacelike (no causal connection)

TRAP IB formula: $(c\Delta t)^2 - (\Delta x)^2$. Some books use the opposite sign. Always use the IB convention: **time first, minus space squared**.

Invariant Quantities — Same in Every Frame

All inertial observers agree on:
- the **space-time interval** $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$
- the **proper time** Δt_0 (events at the same place)
- the **proper length** L_0 (object at rest)
- the **rest mass** m_0 (mass in the object's rest frame — does not change with the observer)

Also invariant: the speed of light c and electric charge.

FROM THE PHOTON QUESTION BANK

Write the interval as $(c\Delta t)^2 - (\Delta x)^2$ with ct and x in the same units; read the sign to classify ($+$ timelike, 0 lightlike, $-$ spacelike). Asked for it "in another frame"? State it is *unchanged*.

§5 — Space-Time Diagrams

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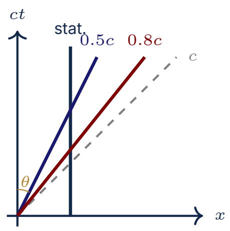
World Lines

- **Axes:** horizontal $= x$, vertical $= ct$
- **Stationary:** vertical world line
- **Moving at v:** $\tan\theta = v/c$ from ct-axis
- **Light:** $45°$ line ($\tan\theta = 1$, $v = c$)
- All world lines must be between vertical and $45°$



S and S′ on Same Diagram

S′ moves at v relative to S:

- ct' axis tilts **towards** $45°$ line by θ, where $\tan\theta = v/c$
- x' axis tilts **towards** $45°$ line by same θ (symmetric)
- Scales on primed axes are **NOT equal** to unprimed — unit intervals are larger
- Lines of constant $(\Delta s)^2$ (hyperbolas) calibrate the axes

NOTE Simultaneity: S-simultaneous events lie on horizontal lines. S′-simultaneous events lie on lines parallel to the x' axis. These are different lines if $v \neq 0$.

Proper Time Along a World Line, Curved Lines & the Twin Paradox

Proper time of a leg:

$$\Delta t_0 = \frac{\sqrt{(c\Delta t)^2 - (\Delta x)^2}}{c}$$

Add legs for a multi-stage journey. The **straight (inertial) world line always has the greatest proper time**; a **curved** (accelerated) world line records *less*.

Twin paradox: traveller flies out and back; each expects the other to be younger. Resolution: only the *travelling* twin accelerates (a *kink* in the world line, switching frames), so the situation is *not* symmetric — the Earth twin ages more. No contradiction.

§6 — Muon Decay & Experimental Evidence

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Muon Decay Argument

Earth frame: muon travels at $\sim 0.98c$; lifetime is dilated to $\gamma\tau_0$. Distance $= v \cdot \gamma\tau_0 \gg v\tau_0$.

Muon frame: muon is at rest; atmosphere length-contracts. Distance to Earth $= L_0/\gamma$.

Both frames agree: the muon reaches sea level.

Summary: What is Proper?

Quantity	Proper	Measured by
Time	Δt_0	moving clock (at both events)
Length	L_0	rest frame of object

Remember: proper time is **shortest**; proper length is **longest**.

TRICK IB always asks you to explain from *both* frames. In Earth frame = time dilation. In muon frame = length contraction. Mention both for full marks.

TRAP Do NOT confuse "proper" with "correct". All measurements are equally valid. "Proper" simply means "measured in the object's own rest frame".