PHOTON ACADEMY

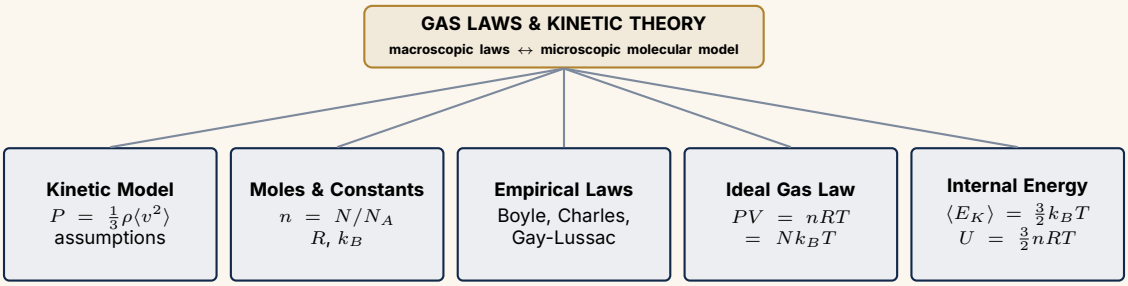
Topic B.3

GAS LAWS

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered	Exam Relevance
Pressure, Temperature & the Mole	P1 & P2 (with & without GDC)
Boyle, Charles & Gay-Lussac	pV/T = constant
Ideal Gas Law $pV = nRT$	$pV = nRT$ calculations
Kinetic Theory of Gases	$p = \frac{1}{3}\rho\langle c^2\rangle$
Internal Energy of an Ideal Gas	$E_K = \frac{3}{2}kT$

The Big Picture — How B.3 Fits Together



§1 — Pressure & Kinetic Model

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Key Formulae

Density: $\rho = \dfrac{m}{V}$

Pressure: $P = \dfrac{F}{A}$

Absolute pressure: $P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}$

Pressure from KE (kinetic model): $P = \dfrac{1}{3}\rho\langle v^2\rangle$

rms speed: $v_{\text{rms}} = \sqrt{\langle v^2\rangle} = \sqrt{\dfrac{3k_BT}{m}}$

Density & Pressure Facts

- Both scalars. Water: $\rho = 1\,\text{g/cm}^3 = 1000\,\text{kg/m}^3$.
- $1\,\text{Pa} = 1\,\text{N/m}^2$; atmospheric $P \approx 10^5$ Pa.
- Pressure at a point acts **equally in all directions**.

TRAP Use **absolute** pressure in $PV = nRT$. A gauge reading is $P - P_{\text{atm}}$; add $\approx 10^5$ Pa back first.

FROM THE PHOTON QUESTION BANK

$P = F/A$ **traps:** use the *actual* contact area (a thin wall's cross-section, not the full outer area). **Density → spacing:** molecular spacing $\propto \rho^{-1/3}$ (cube root of a density ratio).

Kinetic Model Assumptions

1. Large number of identical particles in constant **random motion**
2. Particle volume **negligible** vs container
3. Collisions **perfectly elastic** (KE conserved)
4. **No intermolecular forces** between particles
5. Collision time **negligible** vs time between collisions
6. All directions of motion equally probable

TRICK The factor $\frac{1}{3}$ in $P = \frac{1}{3}\rho v^2$ comes from 3D random motion. One-third of molecules contribute to each direction.

TRICK **Two levels of description:** macroscopic (P, V, T, n; the gas laws) and microscopic (molecular speeds & KE). $P = \frac{1}{3}\rho\langle v^2\rangle$ combined with $PV = Nk_BT$ links them.

TRAP Pressure = force perpendicular to area only. A parallel force creates no pressure.

§2 — Amount of Substance

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Key Formulae

Amount of substance: $n = \dfrac{N}{N_A}$

Mass–moles relation: $n = \dfrac{m}{M_r}$

Number from mass: $N = \dfrac{mN_A}{M_r}$

Boltzmann constant: $k_B = \dfrac{R}{N_A} = 1.38 \times 10^{-23}\,\text{J/K}$

Constants

$N_A = 6.02 \times 10^{23}\,\text{/mol}$ (Avogadro constant)
$R = 8.31\,\text{J/(mol K)}$ (gas constant)
$k_B = 1.38 \times 10^{-23}\,\text{J/K}$ (Boltzmann)

NOTE To find which gas sample has the most molecules from equal masses: the gas with the smallest molar mass M_r has the most moles and hence the most molecules.

§3 — Empirical Gas Laws

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Three Gas Laws

Law	Constant	Relation	Form
Boyle	T	$P \propto 1/V$	$P_1V_1 = P_2V_2$
Charles	P	$V \propto T$	$V_1/T_1 = V_2/T_2$
Gay-Lussac	V	$P \propto T$	$P_1/T_1 = P_2/T_2$

TRAP Temperature **must** be in kelvin: $T(\text{K}) = \theta(^\circ\text{C}) + 273$. Never substitute °C directly.

TRICK Combined law for any process: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$. Cancel whichever variable is constant.

Molecular Explanations

$\downarrow V$ **at const** $T \Rightarrow \uparrow P$**:** molecules hit walls more frequently (shorter distance between collisions)

$\uparrow T$ **at const** $V \Rightarrow \uparrow P$**:** greater speed $\Rightarrow$ more collisions per second & greater momentum transfer

$\uparrow T$ **at const** $P \Rightarrow \uparrow V$**:** greater speed $\Rightarrow$ volume must expand to restore collision rate

Absolute Zero & Experiments

- P (or V) vs *Celsius* θ is a straight line extrapolating to $\theta = -273\,^\circ$C = absolute zero.
- P (or V) $\propto$ **kelvin** T only ($T = \theta + 273$).
- **Const** V**:** rigid flask in bath (P vs T). **Const** P**:** open capillary + acid bead (V vs T). **Const** T**:** oil column + pump (P vs V).

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Convert °C→K *every time*. On a p vs θ plot (const V): gradient $= nR/V$; a line missing the origin flags **systematic error**.

§4 — Ideal Gas Law

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Ideal Gas Law — Three Forms

Molar form: $PV = nRT$

Particle form: $PV = Nk_BT$

Combined states: $P_1V_1/T_1 = P_2V_2/T_2$

Equivalence: $nR = Nk_B \Leftrightarrow k_B = R/N_A$

TRICK Use $PV = nRT$ when given mass or moles. Use $PV = Nk_BT$ when given number of particles. Either form gives identical results.

TRAP $PV/T = nR =$ constant only for **fixed mass**. If gas is added or removed, n changes.

P-V Diagram Features

- Isothermal = hyperbola: $PV =$ const
- Higher curve = higher T
- Isobaric: horizontal line (constant P)
- Isochoric: vertical line (constant V)
- Straight line through origin: P vs $1/V$ at const T

§5 — Internal Energy

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Key Formulae

PV from KE: $PV = \frac{2}{3}N\langle E_K \rangle$

Average KE per molecule: $\langle E_K \rangle = \frac{3}{2}k_B T$

Internal energy (monatomic): $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$

rms speed: $v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$

FROM THE PHOTON QUESTION BANK
Gradient of a U vs T line $= \frac{3}{2}nR \Rightarrow$ read off n. Same T $\Rightarrow$ same $\langle E_K \rangle$, so the heavier molecule is **slower**.

TRICK $\langle E_K \rangle = \frac{3}{2}k_B T$ depends **only on temperature**, regardless of gas type. Equal temperatures $\Rightarrow$ equal average kinetic energies.

TRICK $U =$ **KE** $+$ **PE, but ideal gas PE** $= 0$ (no intermolecular forces). So $U = \frac{3}{2}nRT$ is *all* kinetic and depends on T alone — raising U *is* raising T.

NOTE For monatomic ideal gas: U = kinetic energy only (no PE, no rotation/vibration). $U \propto T$ exactly.

TRAP Heavier molecules at the same T move **slower**, not faster. From $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$: larger $m \Rightarrow$ smaller v_{rms}.

§6 — Ideal vs Real Gases

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Comparison Table

	Ideal	Real
Molecular vol	negligible	finite
Inter-mol forces	none	attractive/repulsive
$PV = nRT$	exact	approximate
$Z = PV/nRT$	$= 1$	$\neq 1$
Internal energy	KE only	KE + PE
Condensation	never	possible

Ideal Approx. Valid When

- **Low pressure** — molecules far apart; volume and forces negligible
- **High / moderate temperature** — KE $\gg$ intermolecular PE; far from condensation
- Far from boiling point of gas

TRAP At high P: $Z > 1$ (repulsive forces, finite volume dominate). At low T / moderate P: $Z < 1$ (attractive forces pull molecules together, reducing pressure).

TRICK On a real-gas P-V diagram: the flat region of an isotherm (constant P) represents a **phase change** (condensation). Ideal gas isotherms are smooth hyperbolas with no flat region.