Topic C.1

SIMPLE HARMONIC MOTION

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered	**Exam Relevance**
The SHM condition $a = -\omega^2 x$	P1 & P2 (SL + HL)
Period of spring & pendulum	$T = 2\pi\sqrt{m/k},\ \sqrt{l/g}$
Energy in SHM	$E \propto x_0^2$ (HL)
Phase & equations of motion	$x = x_0 \sin(\omega t + \varphi)$ (HL)
Graphs of x, v, a	Read ω off an a–x graph

The Big Picture — How C.1 Fits Together

SIMPLE HARMONIC MOTION
$a = -\omega^2 x$: a restoring force $\propto$ −displacement

- **Defining Eqn**: $a = -\omega^2 x$; $F = -kx$
- **Period & Freq.**: $T = \frac{2\pi}{\omega}$; $\omega = \sqrt{k/m}, \sqrt{g/L}$
- **Energy**: KE ↔ PE; $E_T = \frac{1}{2}m\omega^2 x_0^2$
- **Equations (HL)**: $x = x_0 \sin(\omega t + \varphi)$; $v = \omega x_0 \cos(\cdot)$
- **Graphs**: x, v, a phase relations

§1 — Conditions & Defining Equation

C.1 SL–HL

Defining Equation of SHM

$$a = -\omega^2 x$$

Acceleration **proportional** to displacement; always **directed towards** equilibrium. ω = angular frequency ($\mathrm{rad\,s^{-1}}$).

TRICK If an exam question gives you an a-x graph that is a straight line through the origin with *negative* gradient, it is SHM. The gradient $= -\omega^2$.

TRICK **Two ways to spot SHM:** kinematic ($a = -\omega^2 x$, an a–x line of gradient $-\omega^2$) OR dynamic (a restoring force $F = -kx$). Same thing — $\omega^2 = k/m$.

TRAP The acceleration is **not** constant in SHM — it varies with displacement. Never apply SUVAT equations to SHM.

FROM THE PHOTON QUESTION BANK
Read the gradient of an a–x graph, set $|\text{gradient}| = \omega^2$, then convert with $\omega = 2\pi f$ to find frequency or period.

Key Positions

Position	$\lvert a \rvert$	$\lvert v \rvert$
Equilibrium ($x = 0$)	0	maximum
Amplitude ($x = \pm x_0$)	maximum	0

§2 — Period, Frequency, Angular Frequency

C.1 SL–HL

Period–Frequency Relations

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \qquad f = \frac{\omega}{2\pi} \qquad \omega = 2\pi f$$

Mass-Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Independent of g and amplitude.
$k_{\text{parallel}} = nk$; $k_{\text{series}} = k/n$

Simple Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Independent of mass and amplitude.
Valid for $\theta_{\max} \lesssim 10°$ only.

TRICK To compare periods: write $T \propto \sqrt{m/k}$ or $T \propto \sqrt{l/g}$, then use ratios. No need to calculate T numerically if the question asks "by what factor does T change."

TRAP A mass-spring system taken to the Moon has the **same** period (no g in formula). A pendulum on the Moon has a **longer** period (smaller $g \Rightarrow$ larger T).

NOTE Springs in parallel: $k_{\text{eff}} = 2k$ (stiffer). Springs in series: $k_{\text{eff}} = k/2$ (weaker). A mass between two identical springs each of constant k behaves as if $k_{\text{eff}} = 2k$.

FROM THE PHOTON QUESTION BANK

Know which variable each period depends on: the pendulum ($T \propto \sqrt{l/g}$) is mass-independent; the spring ($T \propto \sqrt{m/k}$) is g-independent. Take ratios so the shared unknown cancels.

§3 — Energy in SHM

C.1 SL (qualitative), HL (quantitative)

Qualitative (SL + HL)

E_K is maximum at equilibrium; E_P is maximum at amplitude. $E_T = E_K + E_P =$ constant (no damping). KE and PE exchange 4 times per cycle.

Quantitative (HL only)

$$E_T = \tfrac{1}{2}m\omega^2 x_0^2 \qquad E_P = \tfrac{1}{2}m\omega^2 x^2 \qquad E_K = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$

$$v = \pm\omega\sqrt{x_0^2 - x^2} \qquad v_{\max} = \omega x_0$$

TRICK When $E_K = E_P$: each equals $\frac{1}{2}E_T$, so $x = x_0/\sqrt{2} \approx 0.71x_0$. This occurs 4 times per cycle.

TRICK **Two ways to get speed:** from position $v = \pm\omega\sqrt{x_0^2 - x^2}$ (energy) OR from time $v = \omega x_0 \cos(\omega t + \varphi)$ (HL). Max speed ωx_0 at $x = 0$.

TRAP $E_T \propto x_0^2$. Doubling amplitude **quadruples** total energy. Do not confuse "E_T is constant during oscillation" (at fixed amplitude) with what happens when amplitude changes.

FROM THE PHOTON QUESTION BANK

Total energy scales with m, (amplitude)^2 and (frequency)^2. Max PE $= E_T = \frac{1}{2}kx_0^2$ is mass-independent — read x_0 off a KE–x graph and sketch the complementary PE parabola.

§4 — Phase Angle & Equations of Motion (HL)

C.1 HL

General Equations (HL)

$$x = x_0 \sin(\omega t + \varphi) \qquad v = \omega x_0 \cos(\omega t + \varphi)$$

φ: initial phase angle (rad). Set by initial conditions at $t = 0$.

$\varphi = \pi/2$ starts at $+x_0$ (cosine form)
$\varphi = 0$ starts at equilibrium, moving positive
$\varphi = -\pi/2$ starts at $-x_0$

TRICK The phase difference between x and v is $\pi/2$ rad. The phase difference between x and a is π rad (anti-phase). These are independent of ω or amplitude.

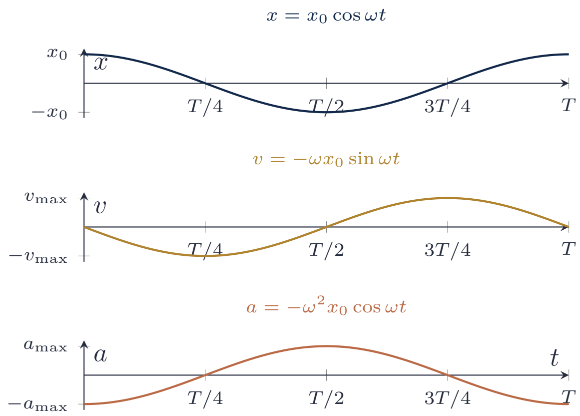
TRICK **sine or cosine?** Starts at equilibrium ($x = 0$ at $t = 0$) $\Rightarrow$ use $x_0 \sin(\omega t)$; starts at amplitude $\Rightarrow$ use $x_0 \cos(\omega t)$. Read the "at $t = 0$ clause first.

TRAP Radians only! Phase angles and ωt must be in radians. Never mix degrees and radians in a single calculation.

NOTE $v = \pm\omega\sqrt{x_0^2 - x^2}$ is derived from the energy equation and is often faster to use than the full sinusoidal expression when only speed (not direction) is required.

§5 — Graphs Summary

C.1 SL–HL



NOTE x and a are in anti-phase ($\Delta\phi = \pi$). v leads x by $\pi/2$. v and a have a phase difference of $\pi/2$.