Topic D.1

GRAVITATIONAL FIELDS

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered

Kepler's laws & $F = GMm/r^2$
Field strength g
Orbital motion & Kepler III
Potential & escape speed (AHL)
Energy of a satellite (AHL)

Exam Relevance

P1 & P2 (SL + HL)
Centre-to-centre r
$v = \sqrt{GM/r}$
$V_g = -GM/r$
$E_T = -GMm/2r$

The Big Picture — How D.1 Fits Together

GRAVITATIONAL FIELDS
one source law $F = GMm/r^2$
generates force, field, energy & orbits

- **Newton & Kepler**: $F = \frac{GMm}{r^2}$, $T^2 \propto r^3$
- **Field g**: $g = \frac{F}{m} = \frac{GM}{r^2}$, vector, adds
- **Orbits**: $v = \sqrt{\frac{GM}{r}}$, weightless = free fall
- **Potential (AHL)**: $V = -\frac{GM}{r}$, scalar, adds
- **Energy (AHL)**: $E_T = -\frac{GMm}{2r}$, escape, drag

§1 — Kepler's Laws & Newton's Law of Gravitation

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Kepler's Three Laws

1st Law (Ellipses): Every planet moves in an elliptical orbit with the Sun at one focus.
2nd Law (Equal Areas): A line from planet to Sun sweeps equal areas in equal times. $\Rightarrow$ Faster at perihelion, slower at aphelion. Consequence of conservation of angular momentum.
3rd Law (Harmonic): $T^2 \propto r^3$ for all bodies orbiting the same central mass.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \frac{4\pi^2}{GM}$$

Newton's Law of Gravitation: $F = \dfrac{GMm}{r^2}, \quad G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

TRAP r is always centre-to-centre distance. For a satellite at height h above a planet of radius R: $r = R + h$, NOT h.

TRICK Inverse-square law quick check: if $r \to 2r$, then $F \to F/4$. If $r \to 3r$, then $F \to F/9$.

TRICK **Two ways to see g:** (1) force per unit mass you'd *measure*, $g = F/m$; (2) a property of space *sourced* by M, $g = GM/r^2$ — present even with no test mass there.

FROM THE PHOTON QUESTION BANK
Exam-style questions love ratios: given a mass ratio and a *diameter* ratio, halve to get radii before $g \propto M/r^2$. With "equal density" spheres, first convert $m = \rho \cdot \frac{4}{3}\pi r^3$. Do Kepler-3 as $T_1^2/r_1^3 = T_2^2/r_2^3$ — never plug in G.

§2 — Gravitational Field Strength

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Definition: $g = \dfrac{F}{m}$ [N/kg] = [m/s^2]

From Newton's law: $g = \dfrac{GM}{r^2}$ (at distance r from centre of mass M)

At surface: $g_{\text{surface}} = \dfrac{GM}{R^2}$

NOTE g is a vector — it points **toward** the mass. Field lines point radially inward for a sphere. Between two masses, fields oppose each other.

Neutral Point

Between masses M_1 and M_2 separated by D, the neutral point ($g_{\text{net}} = 0$) is at distance d from M_1:

$$\frac{d}{D-d} = \sqrt{\frac{M_1}{M_2}} \quad \Rightarrow \quad d = \frac{D}{1 + \sqrt{M_2/M_1}}$$

Always **closer to the smaller mass**.

TRAP The neutral point is where $\vec{g} = 0$, NOT where $V_g = 0$. Potential is never zero between two masses.

§3 — Orbital Motion & Kepler's Third Law Derivation

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Key Derivation — Learn This!

Equate gravitational force = centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v^2 = \frac{GM}{r} \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{GM}{r}}}$$

Substitute $v = \frac{2\pi r}{T}$:

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \quad \Rightarrow \quad \boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$

TRICK Orbital speed $\propto 1/\sqrt{r}$: closer satellites orbit faster. Period $\propto r^{3/2}$: closer satellites have shorter periods. Satellite mass cancels out of everything.

TRAP Do not confuse **orbital speed** $v = \sqrt{GM/r}$ (stay in orbit) with **escape speed** $v_{\text{esc}} = \sqrt{2GM/R}$ (leave the field). At the same r: $v_{\text{esc}} = \sqrt{2}\, v_{\text{orbital}}$.

FROM THE PHOTON QUESTION BANK

"Show that" derivations must start from $GMm/r^2 = mv^2/r$. Geostationary items fix $T = 24\,\text{h}$ — convert to seconds, solve $T^2 = \frac{4\pi^2}{GM} r^3$ for r, then subtract R_E for altitude. A quick dimensional check on $\sqrt{GM/r}$ earns the mark.

§4 — Weightlessness

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NOTE Apparent weightlessness occurs when the astronaut and spacecraft are **both in free fall** — they have the same acceleration, so no contact (normal) force exists between them.

TRAP NEVER say "there is no gravity in space." At ISS altitude ($\sim 400\,\text{km}$), $g \approx 8.7\,\text{m/s}^2$ — only 11% less than at the surface.

§5 — Gravitational Potential Energy & Potential (AHL)

D.1 AHL

NOTE Near a surface, $\Delta E_p = mg(h_2 - h_1)$ — but only for small heights (assumes g constant, zero taken at the surface). The fundamental zero is at infinity, giving $E_p = -GMm/r$.

Gravitational PE: $E_p = -\dfrac{GMm}{r}$ (always negative; zero at $r = \infty$)

Gravitational Potential: $V_g = \dfrac{E_p}{m} = -\dfrac{GM}{r}$ [J/kg]

Work done: $W = m\,\Delta V_g = m(V_f - V_i)$

TRICK Potential is a SCALAR — add potentials from multiple masses **algebraically** (not as vectors). This is unlike field strength g which requires vector addition.

TRICK **Two ways to see potential:** (1) energy *per kg* of test mass, $V = E_p/m$; (2) the depth of the gravity well at that point. Compute $V = -GM/r$ once, then any object's energy is $E_p = mV$.

NOTE Gravitational field is conservative: work depends only on start/end positions, not path. Work around a closed loop = 0.

§6 — Potential Gradient, Equipotentials & Escape Speed (AHL)

D.1 AHL

Potential gradient: $g = -\dfrac{\Delta V_g}{\Delta r}$ (g = negative slope of V_g vs r graph)

Equipotential Surfaces

- Surfaces where V_g = constant
- Always $\perp$ to field lines
- No work done moving along an equipotential ($\Delta V_g = 0$)
- For a point mass: concentric spheres
- Closer spacing = stronger field

Escape speed: $v_{\text{esc}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{2gR} = \sqrt{2} \times v_{\text{orbital at surface}}$

TRAP Escape speed does NOT depend on the mass of the escaping object. It also does not depend on direction of launch.

FROM THE PHOTON QUESTION BANK

Derive v_{esc} from $\frac{1}{2}mv^2 = GMm/R$. Density form: sub $M = \frac{4}{3}\pi R^3 \rho$ to get $v_{\text{esc}} \propto R\sqrt{\rho}$ for ratio questions. If a graph gives surface potential V, then $v_{\text{esc}} = \sqrt{2|V|}$.

§7 — Energy of an Orbiting Satellite (AHL)

D.1 AHL

Satellite Energy at Orbital Radius r

Kinetic energy: $E_K = \dfrac{GMm}{2r}$ (positive)

Potential energy: $E_p = -\dfrac{GMm}{r}$ (negative)

Total energy: $E_T = -\dfrac{GMm}{2r}$ (negative = bound)

Key: $E_K = -\frac{1}{2}E_p, \quad E_T = -E_K = \frac{1}{2}E_p$

Work to change orbit: $W = \Delta E_T = \dfrac{GMm}{2}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

TRICK The "drag paradox": atmospheric drag causes a satellite to spiral inward and **speed up**. As r decreases, $v = \sqrt{GM/r}$ increases. E_K increases, E_p decreases (more negative), E_T decreases (more negative — energy lost to heat).

TRAP Do not say "drag slows the satellite." The instantaneous effect is deceleration, but the satellite drops to a lower, faster orbit. Net effect: speed increases.