Topic E.3

RADIOACTIVE DECAY

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

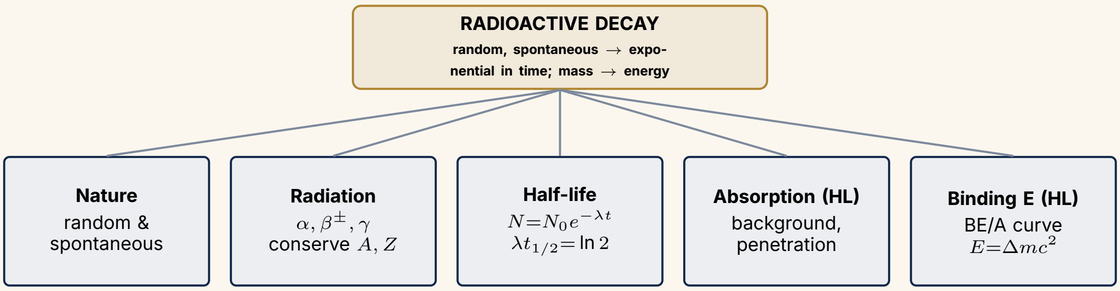
Topics Covered

Nature of decay
Types of radiation (α, β, γ)
Half-life & activity
Background & attenuation (HL)
Binding energy & $E = mc^2$ (HL)

Exam Relevance

P1 & P2 (SL + HL)
Penetration & ionising power
$N = N_0 e^{-\lambda t}$
$I = I_0 e^{-\mu x}$
BE/A peaks at ^{56}Fe

The Big Picture — How E.3 Fits Together



§1 — Nature of Radioactive Decay

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Core Properties

- **Random:** Cannot predict *which* nucleus decays next, or *when*.
- **Spontaneous:** Not affected by temperature, pressure, or chemical state.
- **Nuclide notation:** $^{A}_{Z}\text{X}$ where A = nucleon number, Z = proton number.

NOTE Randomness is a property of individual nuclei. Large numbers of nuclei behave statistically — hence a well-defined half-life.

§2 — Types of Radiation

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Radiation Summary Table

Type	Identity	Charge	Ionising	Stopped by
α	$^{4}_{2}\text{He}$ nucleus	$+2e$	Highest	Paper / cm of air
β^-	Electron e^-	$-e$	Medium	Few mm Al
β^+	Positron e^+	$+e$	Medium	Few mm Al
γ	Photon (EM)	0	Lowest	Several cm Pb

Alpha decay:

$$^{A}_{Z}\text{X} \to {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$$

Beta-minus decay:

$$^{A}_{Z}\text{X} \to {}^{A}_{Z+1}\text{Y} + e^- + \bar{\nu}_e$$

Beta-plus decay: $^{A}_{Z}\text{X} \to {}^{A}_{Z-1}\text{Y} + e^+ + \nu_e$

TRICK Conservation check for all decays: top numbers (A) must balance; bottom numbers (Z) must balance. Neutrinos: $A = 0$, $Z = 0$.

TRAP β^- and β^+ energies are **continuous spectra** (shared with neutrino/antineutrino). Gamma energy is **discrete** (nuclear energy levels). α energy is also discrete. This was the key evidence for neutrino existence.

§3 — Half-Life, Activity and Decay Laws

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Key Formulae

Activity: $A = \lambda N$

Decay constant: $\lambda = \dfrac{\ln 2}{t_{1/2}}$

Number of nuclei: $N = N_0\, e^{-\lambda t}$

Activity (time): $A = A_0\, e^{-\lambda t}$

After n half-lives: $N = \dfrac{N_0}{2^n}, \quad A = \dfrac{A_0}{2^n}$

Unit of activity: 1 Bq $= 1$ decay s^{-1}

Half-life from graph: $t_{1/2} = t_2 - t_1$ (choose any N, $N/2$)

TRICK To find half-life from a graph: pick *any* starting N value, halve it, read across to the curve, then down to get the time difference. Do it twice to confirm.

TRICK **Two ways to count decay:** whole number of half-lives → just halve, $N = N_0/2^n$; awkward time → use $N = N_0 e^{-\lambda t}$. Same curve; halving is faster when it applies.

TRAP Always subtract background before analysis: $A_{\text{true}} = A_{\text{measured}} - A_{\text{background}}$. Forgetting background is the most common experimental error.

NOTE λ = probability of decay **per unit time** per nucleus. **Not** the number of decays. (exam-style, Answer: D)

FROM THE PHOTON QUESTION BANK

Common exam-style tasks: convert a sample *mass* to nuclei ($n\,N_A$) then use $A = \lambda N$; and for two nuclides over one time, count each half-life set separately before applying $(1/2)^n$.

§4 — HL: Background Radiation & Absorption

Topic E.3 (HL)

Background Radiation Sources

- **Natural (largest):** Radon gas (from soil/rocks), cosmic rays, ^{40}K in food/body, gamma from building materials.
- **Artificial:** Medical X-rays, nuclear power, weapons testing fallout.

Absorption Mechanisms (HL)

α	Direct ionisation (loses energy in air collisions)	Paper/skin
$\beta^{\pm}$	Ionisation + Bremsstrahlung (braking radiation)	Al (mm)
γ	Photoelectric effect, Compton scattering, pair production	Pb (cm)

Gamma attenuation: $I = I_0\, e^{-\mu x}$ (μ = linear attenuation coefficient)

§5 — HL: Binding Energy & $E = mc^2$

Topic E.3 (HL)

Binding Energy Formulae

Mass defect: $\Delta m = Z m_p + (A - Z) m_n - m_{\text{nuc}}$

Binding energy: $E_B = \Delta m \cdot c^2$

BE per nucleon: E_B/A

Unit conversion: 1 u $c^2 = 931.5$ MeV

Energy released: $Q = E_B$(products) $-$ E_B(reactants)

The BE/A Curve — Key Facts

- **Peak** at $^{56}_{26}$Fe: $E_B/A \approx 8.79$ MeV — most stable nucleus.
- **Fission**: heavy nucleus ($A \approx 235$) splits into medium fragments (higher E_B/A) ⇒ energy released.

- **Fusion**: light nuclei ($A < 10$) combine to form heavier products (higher E_B/A) $\Rightarrow$ energy released.
- ^{56}Fe can neither fission nor fusion to release energy — it is at the peak.

TRICK Energy released = (total BE of products) $-$ (total BE of reactants). If products have **higher** total BE, energy is released (not absorbed).

TRICK **Two ways to see energy release:** (1) move *towards* the BE/A peak (Fe-56) — fusion climbs from the left, fission drops from the right; (2) products lose mass, $E = \Delta mc^2$. Same fact.

TRAP Mass defect uses the **nuclear** mass, not atomic mass. If atomic masses are given, subtract $Z \times m_e$ from each, or be consistent (the electron masses cancel in the difference if you use atomic masses throughout).

FROM THE PHOTON QUESTION BANK

Total binding energy $=$ (BE per nucleon from the graph) $\times A$ — forgetting $\times A$ is the classic slip. "Specific energy" of a fuel divides the released energy by the *total reactant mass*, not the mass defect.

§6 — Fission, Fusion, Forces & Uses

Topic E.3 (SL–HL)

Fission and Fusion

- **Fission:** large nucleus splits, e.g. ${}^1_0$n$+{}^{235}_{92}$U $\rightarrow {}^{141}_{56}Ba+{}^{92}_{36}Kr+3\,{}^1_0$n. Spare neutrons $\Rightarrow$ **chain reaction**.
- **Fusion:** small nuclei join, e.g. ${}^2_1$H $+ {}^3_1$H $\rightarrow {}^4_2$He $+ {}^1_0$n. Fuels the stars.
- Both release energy by moving *towards* the ^{56}Fe peak (products have higher E_B/A; mass lost $\rightarrow$ energy).

Fundamental Forces

- **Strong:** very strong, very short range; binds nucleons against proton repulsion; heavy nuclei need extra neutrons.
- **Weak:** responsible for beta decay; involves the light particles (e, positron, neutrino).
- **Gravity** & **Electromagnetic:** infinite range; EM & weak unify as the *electroweak* force.

Uses of Radioactivity

- **Medical:** γ tracers; radiotherapy (critical dose); γ sterilisation.
- **Industrial:** γ for pipe leaks (short $t_{1/2}$); thickness gauges (β paper, γ metal; long $t_{1/2}$).
- **Dating:** ^{14}C for once-living matter; ^{238}U, ^{40}K for rocks.

TRICK Choose the isotope by penetration + half-life: γ to escape body/pipe/metal, β for thin gauging; *short* $t_{1/2}$ for tracers/leaks, *long* for steady industrial sources.

§7 — HL: Nuclear Stability & Decay Maths

Topic E.3 (HL)

Band of Stability

- Small nuclei: $N \approx Z$; large nuclei: $N > Z$ (more neutrons dilute repulsion).
- **Above** band (too many n) $\Rightarrow \beta^-$; **below** band (too few n) $\Rightarrow \beta^+$.
- Strong-force evidence: high-energy deviations from Rutherford scattering.

Decay Maths (HL)

Random law: $\dfrac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$

Solution: $N = N_0 e^{-\lambda t}$

Linearised: $\ln N = \ln N_0 - \lambda t$

Gradient of $\ln N$ vs t: $-\lambda$

Half-life relation: $\lambda t_{1/2} = \ln 2$

Decay probability: $P = \lambda \Delta t$

TRICK To get λ experimentally: plot $\ln A$ vs t (straight line), take the gradient, $\lambda = -$gradient.

TRAP Beta energy is a **continuous** spectrum because the antineutrino shares the fixed released energy — α and γ are discrete (nuclear energy levels).