Topic E.5

FUSION & STARS

Every Formula, Trick & Trap on a Page
& IB-Style Quick Reference

Topics Covered	Exam Relevance
Nuclear fusion	P1 & P2 (SL + HL)
Stellar radiation laws	Wien & Stefan–Boltzmann
The HR diagram	Main sequence, giants, dwarfs
Stellar evolution	White dwarf vs supernova
Distances: parallax & candles (HL)	$d(\text{pc}) = 1/p$

The Big Picture — How E.5 Fits Together

FUSION & STARS
fusion powers stars; their light tells us T, L, R and fate

- **Fusion** — light nuclei ↑ BE/A
- **Radiation** — $L=4\pi R^2\sigma T^4$, $b=L/4\pi d^2$
- **HR Diagram** — L vs T, main sequence
- **Evolution** — giant → WD/NS/BH; limits
- **Distances (HL)** — M–L, Jeans, standard candles

§1 — Nuclear Fusion E.5 SL/HL

Key Fusion Reactions

Proton-proton (net): $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu_e + 2\gamma$

D-T fusion: ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + 17.6\,\text{MeV}$

D-D fusion: ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{H} + {}^1_1\text{p} + 4.0\,\text{MeV}$

TRICK Mass *decreases* in fusion (binding energy per nucleon increases). Use $E = \Delta mc^2$ for energy released.

TRICK **Two ways to see fusion energy:** climb the BE/A curve (product more tightly bound) *or* mass defect $E = \Delta mc^2$ — same energy, mirror image of fission.

TRAP Specific energy of fusion (energy per kg of fuel) is *higher* than fission because H nuclei are much lighter than U-235.

Conditions for Fusion

- **High temperature** ($\sim 10^7$ K): gives protons enough KE to overcome Coulomb repulsion
- **High density**: increases collision frequency $\Rightarrow$ higher reaction rate
- Both are needed — don't state just one!

NOTE The Sun's core: $T = 1.5 \times 10^7$ K, $\rho = 1.6 \times 10^5$ kg/m^3.

Hydrogen Burning & Nucleosynthesis

- Star's energy = fusion of H → He (a *nuclear*, not chemical, process)
- **p–p chain**: Sun-like (small/medium) stars
- **CNO cycle**: hotter, higher-mass stars; C, N, O are *catalysts*
- Both give net $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu$
- **Nucleosynthesis**: building heavier nuclei by fusion in stars

NOTE Sun loses mass $\sim 4 \times 10^9$ kg/s and radiates $\sim 10^{26}$ J each second. Core temperature comes from gravitational PE → KE as the gas cloud collapsed.

FROM THE PHOTON QUESTION BANK
Balance fusion equations by conserving nucleon (top) and proton (bottom) numbers; the missing piece is usually a positron/neutrino. Name p–p (Sun) vs CNO (hot massive stars) — both net to $4\,{}^1\text{H} \rightarrow {}^4\text{He}$.

§2 — Stellar Radiation Laws E.5 SL/HL

Key Formulae

Wien's Law: $\lambda_{\text{max}} T = 2.9 \times 10^{-3}\,\text{m K}$

Stefan-Boltzmann: $L = \sigma A T^4 = 4\pi R^2 \sigma T^4$

Apparent Brightness: $b = \dfrac{L}{4\pi d^2}$

Radius Ratio: $\dfrac{R_A}{R_B} = \sqrt{\dfrac{L_A}{L_B}} \cdot \left(\dfrac{T_B}{T_A}\right)^2$

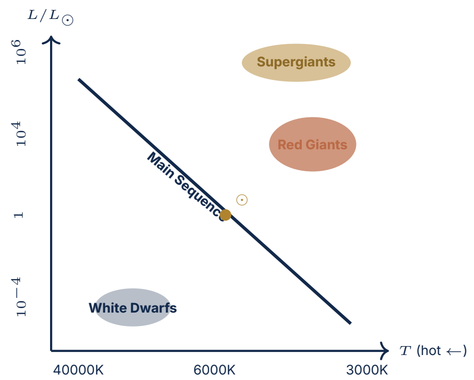
TRICK For ratio problems: $L_A/L_B = (R_A/R_B)^2 (T_A/T_B)^4$ avoids needing σ.

TRAP Wien: units are m·K (NOT m/K). $\lambda_{\text{max}} = 2.9 \times 10^{-3}/T$. Convert to nm: divide metres by 10^{-9}.

FROM THE PHOTON QUESTION BANK
Wien & Stefan–Boltzmann are the two most-tested skills. Ratio of peak wavelengths = *inverse* ratio of temperatures. For $L = 4\pi R^2 \sigma T^4$, equal-L stars give $R \propto 1/T^2$; the T^4 term dominates over radius.

TRAP Temperature axis on HR diagram runs *right to left* (hot stars on the LEFT).

§3 — Hertzsprung-Russell Diagram E.5 SL/HL



NOTE Lines of const R: $L \propto R^2T^4$ so higher-R lines run parallel to main sequence, shifted upper-right.

NOTE **Instability strip**: near-vertical band (~6000–9000 K, above the MS) of pulsating stars (Cepheids).

TRAP A star does NOT slide along the main sequence as it ages — it sits at one fixed point, then *leaves* the MS. Draw tracks that leave, never arrows down the MS.

§4 — Stellar Evolution E.5 SL/HL

Evolution Pathways

Low/medium mass ($\lesssim 8M_\odot$, e.g. Sun):

MS → Red Giant → Planetary Nebula → **White Dwarf**

High mass ($\gtrsim 8M_\odot$):

$$\text{MS} \to \text{Red Supergiant} \to \text{Supernova} \to \begin{cases} \text{Neutron star} \\ \text{Black hole} \end{cases}$$

Mass Limits

- **Chandrasekhar limit** $\approx 1.4\,M_\odot$: max mass of white dwarf (electron degeneracy)
- **Oppenheimer-Volkoff limit** $\approx 3\,M_\odot$: max mass of neutron star (neutron degeneracy)
- Above OV limit ⇒ black hole

TRICK 2-solar-mass star → red giant → planetary nebula → white dwarf. NOT a supernova!

TRAP "Red giant" vs "Red supergiant": giants from low-mass stars; supergiants from high-mass stars. Different endpoints!

Types of Star & Remnant

- **Red giant/supergiant**: large, red ⇒ cool; fuses heavier elements (onion shells → Fe core)
- **White dwarf**: small, white ⇒ hot; no fusion, just cooling
- **Neutron star**: collapsed to neutrons; a spinning one is a **pulsar**
- **Black hole**: escape velocity $> c$; nothing (not even light) escapes

FROM THE PHOTON QUESTION BANK

Pick the fate from initial mass; apply Chandrasekhar ($1.4M_\odot$, e⁻ degeneracy) and Oppenheimer–Volkoff ($3M_\odot$, neutron degeneracy) to the remnant core mass; name the supporting degeneracy pressure.

§5 — HL: Mass-Luminosity & Jeans Criterion E.5 HL

Mass-Luminosity Relation

$$: \quad L \propto M^{3.5} \quad \Rightarrow \quad \frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

MS lifetime: $t \propto M/L \propto M^{-2.5}$

TRICK To find M from L: $M/M_\odot = (L/L_\odot)^{1/3.5} = (L/L_\odot)^{0.286}$.

Jeans Criterion

- Cloud collapses when $|E_{\text{grav}}| > E_{\text{thermal}}$
- Jeans mass $M_J \propto T^{3/2}\,\rho^{-1/2}$
- Lower T or higher ρ ⇒ easier to collapse ⇒ star formation

NOTE Supernovae can trigger nearby cloud collapse by compressing the cloud (increasing ρ).

§6 — HL: Standard Candles & Parallax E.5 HL

Parallax Distance

$$: \quad d\,(\text{pc}) = \frac{1}{p\,('')}$$

1 pc =: 3.09×10^{16} m ≈ 3.26 ly

TRAP Parallax only works to ~1000 pc. Beyond this, p is too small to measure.

Type Ia Supernovae

- White dwarf accretes mass $\rightarrow$ exceeds Chandrasekhar limit $\rightarrow$ thermonuclear explosion
- Always at $\approx 1.4\,M_{\odot}$ $\Rightarrow$ **same peak luminosity every time**
- $L_{\text{peak}} \approx 4 \times 10^{43}$ W $\approx 5 \times 10^{9}\,L_{\odot}$
- Used to measure distances to remote galaxies

TRICK From brightness: $d = \sqrt{L/(4\pi b)}$. Know the formula and use it in both directions.

TRICK **Luminosity vs. brightness:** L is intrinsic (what the star emits); $b = L/4\pi d^2$ is what we receive. Dim-looking can mean far, not faint.

TRAP Type **Ia** (white dwarf) = standard candle. Type **II** (core collapse) = NOT a standard candle (variable luminosity).