



PHOTON ACADEMY

# IB Math AA HL

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Data Booklet - 2021 Syllabus

Every formula and statistical table you'll have access to in the IB AA HL exam - Papers 1, 2 and 3. Re-issued with Photon Academy branding for our students.



# Mathematics: analysis and approaches formula booklet

For use during the course and in the examinations  
First examinations 2021

Version 1.3



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## Prior learning – SL and HL

Area of a parallelogram	$A = bh$ , where $b$ is the base, $h$ is the height
Area of a triangle	$A = \frac{1}{2}(bh)$ , where $b$ is the base, $h$ is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$ , where $a$ and $b$ are the parallel sides, $h$ is the height
Area of a circle	$A = \pi r^2$ , where $r$ is the radius
Circumference of a circle	$C = 2\pi r$ , where $r$ is the radius
Volume of a cuboid	$V = lwh$ , where $l$ is the length, $w$ is the width, $h$ is the height
Volume of a cylinder	$V = \pi r^2 h$ , where $r$ is the radius, $h$ is the height
Volume of a prism	$V = Ah$ , where $A$ is the area of cross-section, $h$ is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$ , where $r$ is the radius, $h$ is the height
Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

## Topic 1: Number and algebra – SL and HL

<b>SL 1.2</b>	<p>The <math>n</math>th term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>	$u_n = u_1 + (n - 1)d$ $S_n = \frac{n}{2}(2u_1 + (n - 1)d); S_n = \frac{n}{2}(u_1 + u_n)$
<b>SL 1.3</b>	<p>The <math>n</math>th term of a geometric sequence</p> <p>The sum of <math>n</math> terms of a finite geometric sequence</p>	$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$
<b>SL 1.4</b>	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where <math>FV</math> is the future value,  <math>PV</math> is the present value, <math>n</math> is the number of years,  <math>k</math> is the number of compounding periods per year,  <math>r\%</math> is the nominal annual rate of interest</p>
<b>SL 1.5</b>	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$
<b>SL 1.7</b>	Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
<b>SL 1.8</b>	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r},  r  < 1$
<b>SL 1.9</b>	Binomial theorem $n \in \mathbb{N}$	$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$ ${}^n C_r = \frac{n!}{r!(n-r)!}$

## Topic 1: Number and algebra – HL only

<b>AHL 1.10</b>	Combinations  Permutations  Extension of binomial theorem, $n \in \mathbb{Q}$	${}^n C_r = \frac{n!}{r!(n-r)!}$ ${}^n P_r = \frac{n!}{(n-r)!}$ $(a+b)^n = a^n \left( 1 + n \left( \frac{b}{a} \right) + \frac{n(n-1)}{2!} \left( \frac{b}{a} \right)^2 + \dots \right)$
<b>AHL 1.12</b>	Complex numbers	$z = a + bi$
<b>AHL 1.13</b>	Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
<b>AHL 1.14</b>	De Moivre's theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

## Topic 2: Functions – SL and HL

<b>SL 2.1</b>	Equations of a straight line	$y = mx + c$ ; $ax + by + d = 0$ ; $y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
<b>SL 2.6</b>	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
<b>SL 2.7</b>	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$
<b>SL 2.9</b>	Exponential and logarithmic functions	$a^x = e^{x \ln a}$ ; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$

## Topic 2: Functions – HL only

<b>AHL 2.12</b>	Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^n a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$ ; product is $\frac{(-1)^n a_0}{a_n}$
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## Topic 3: Geometry and trigonometry – SL and HL

<b>SL 3.1</b>	<p>Distance between two points <math>(x_1, y_1, z_1)</math> and <math>(x_2, y_2, z_2)</math></p> <p>Coordinates of the midpoint of a line segment with endpoints <math>(x_1, y_1, z_1)</math> and <math>(x_2, y_2, z_2)</math></p> <p>Volume of a right-pyramid</p> <p>Volume of a right cone</p> <p>Area of the curved surface of a cone</p> <p>Volume of a sphere</p> <p>Surface area of a sphere</p>	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ $V = \frac{1}{3} Ah, \text{ where } A \text{ is the area of the base, } h \text{ is the height}$ $V = \frac{1}{3} \pi r^2 h, \text{ where } r \text{ is the radius, } h \text{ is the height}$ $A = \pi r l, \text{ where } r \text{ is the radius, } l \text{ is the slant height}$ $V = \frac{4}{3} \pi r^3, \text{ where } r \text{ is the radius}$ $A = 4\pi r^2, \text{ where } r \text{ is the radius}$
<b>SL 3.2</b>	<p>Sine rule</p> <p>Cosine rule</p> <p>Area of a triangle</p>	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $c^2 = a^2 + b^2 - 2ab \cos C; \cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $A = \frac{1}{2} ab \sin C$
<b>SL 3.4</b>	<p>Length of an arc</p> <p>Area of a sector</p>	$l = r\theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$ $A = \frac{1}{2} r^2 \theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$

<b>SL 3.5</b>	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
<b>SL 3.6</b>	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

### Topic 3: Geometry and trigonometry – HL only

<b>AHL 3.9</b>	Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta}$ $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
<b>AHL 3.10</b>	Compound angle identities	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
	Double angle identity for tan	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
<b>AHL 3.12</b>	Magnitude of a vector	$ \mathbf{v}  = \sqrt{v_1^2 + v_2^2 + v_3^2}$ , where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$



## Topic 4: Statistics and probability – SL and HL

<b>SL 4.2</b>	Interquartile range	$IQR = Q_3 - Q_1$
<b>SL 4.3</b>	Mean, $\bar{x}$ , of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$ , where $n = \sum_{i=1}^k f_i$
<b>SL 4.5</b>	Probability of an event $A$	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
<b>SL 4.6</b>	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)P(B)$
<b>SL 4.7</b>	Expected value of a discrete random variable $X$	$E(X) = \sum x P(X = x)$
<b>SL 4.8</b>	Binomial distribution $X \sim B(n, p)$	
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1-p)$
<b>SL 4.12</b>	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

## Topic 4: Statistics and probability – HL only

<b>AHL 4.13</b>	Bayes' theorem	$P(B   A) = \frac{P(B) P(A   B)}{P(B) P(A   B) + P(B') P(A   B')}$ $P(B_i   A) = \frac{P(B_i) P(A   B_i)}{P(B_1) P(A   B_1) + P(B_2) P(A   B_2) + P(B_3) P(A   B_3)}$
<b>AHL 4.14</b>	Variance $\sigma^2$	$\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$
	Standard deviation $\sigma$	$\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$
	Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $\text{Var}(aX + b) = a^2 \text{Var}(X)$
	Expected value of a continuous random variable $X$	$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$
	Variance	$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$
	Variance of a discrete random variable $X$	$\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$
	Variance of a continuous random variable $X$	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

## Topic 5: Calculus – SL and HL

<b>SL 5.3</b>	Derivative of $x^n$	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
<b>SL 5.5</b>	Integral of $x^n$  Area between a curve $y = f(x)$ and the $x$ -axis, where $f(x) > 0$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$  $A = \int_a^b y dx$
<b>SL 5.6</b>	Derivative of $\sin x$  Derivative of $\cos x$  Derivative of $e^x$  Derivative of $\ln x$  Chain rule  Product rule  Quotient rule	$f(x) = \sin x \Rightarrow f'(x) = \cos x$  $f(x) = \cos x \Rightarrow f'(x) = -\sin x$  $f(x) = e^x \Rightarrow f'(x) = e^x$  $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$  $y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$  $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$  $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
<b>SL 5.9</b>	Acceleration  Distance travelled from $t_1$ to $t_2$  Displacement from $t_1$ to $t_2$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$  distance = $\int_{t_1}^{t_2}  v(t)  dt$  displacement = $\int_{t_1}^{t_2} v(t) dt$

<b>SL 5.10</b>	Standard integrals	$\int \frac{1}{x} dx = \ln x  + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
<b>SL 5.11</b>	Area of region enclosed by a curve and $x$ -axis	$A = \int_a^b  y  dx$

## Topic 5: Calculus – HL only

<b>AHL 5.12</b>	Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$
<b>AHL 5.15</b>	Standard derivatives	
	$\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
	$\sec x$	$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
	$\operatorname{cosec} x$	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$
	$\cot x$	$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$
	$a^x$	$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$
	$\log_a x$	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$
	$\arcsin x$	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
	$\arccos x$	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
	$\arctan x$	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

<b>AHL 5.15</b>	Standard integrals	$\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, \quad  x  < a$
<b>AHL 5.16</b>	Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad \text{or} \quad \int u dv = uv - \int v du$
<b>AHL 5.17</b>	Area of region enclosed by a curve and $y$ -axis  Volume of revolution about the $x$ or $y$ -axes	$A = \int_a^b  x  dy$ $V = \int_a^b \pi y^2 dx \quad \text{or} \quad V = \int_a^b \pi x^2 dy$
<b>AHL 5.18</b>	Euler's method  Integrating factor for $y' + P(x)y = Q(x)$	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h, \quad \text{where } h \text{ is a constant (step length)}$ $e^{\int P(x) dx}$
<b>AHL 5.19</b>	Maclaurin series  Maclaurin series for special functions	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$ $e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$